

Spin structure function g_1 at small x and arbitrary Q^2 : Total resummation of leading logarithms vs Standard Approach

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The Standard Approach (SA) for description of the structure function g_1 combines the DGLAP evolution equations and Standard Fits for the initial parton densities. The DGLAP equations describe the region of large Q^2 and large x , so there are not theoretical grounds to exploit them at small x . In practice, extrapolation of DGLAP into the region of large Q^2 and small x is done with complementing DGLAP with special, singular ($\sim x^{-a}$) phenomenological fits for the initial parton densities. The factors x^{-a} are wrongly believed to be of the non-perturbative origin. Actually, they mimic the resummation of logs of x and should be expelled from the fits when the resummation is accounted for. Contrary to SA, the resummation of logarithms of x is a straightforward and natural way to describe g_1 in the small- x region. This approach can be used at both large and small Q^2 where DGLAP cannot be used by definition. Confronting this approach and SA demonstrates that the singular initial parton densities and the power Q^2 -corrections (or at least a sizable part of them) are rather not real physical phenomena but the artefacts caused by extrapolating DGLAP into the small- x region.

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I. INTRODUCTION

The Standard Approach (SA) for description of the structure function g_1 involves the DGLAP evolution equations[1] and Standard Fits[2] for the initial parton densities δq and δg . The fits are defined from phenomenological considerations at $x \sim 1$ and $Q^2 = \mu^2 \sim 1\text{GeV}^2$. The DGLAP equations are one-dimensional, they describe the Q^2 -evolution only, converting δq and δg into the evolved distributions Δq and Δg . They represent g_1 at the region **A**:

$$\mathbf{A:} \quad Q^2 \gg \mu^2, \quad x \lesssim 1. \quad (1)$$

The x -evolution is supposed to come from convoluting Δq and Δg with the coefficient functions C_{DGLAP} . However, in the leading order $C_{DGLAP}^L = 1$; the NLO corrections account for one- or two- loop contributions and neglect higher loops. It is the correct approximation in the region **A** but becomes false in the region **B**:

$$\mathbf{B:} \quad Q^2 \gg \mu^2, \quad x \ll 1 \quad (2)$$

where contributions $\sim \ln^k(1/x)$ are large and should be accounted for to all orders in α_s . C_{DGLAP} do not include the total resummation of leading logarithms of x (LL), so there are not theoretical grounds to exploit DGLAP at small x . However regardless of that, SA extrapolates DGLAP into the region **B**, invoking special fits for δq and δg . A general structure of such fits (see Refs. [2]) is as follows:

$$\delta q = Nx^{-a}\varphi(x) \quad (3)$$

where N is a normalization constant; $a > 0$, so x^{-a} is singular when $x \rightarrow 0$ and $\varphi(x)$ is regular in x at $x \rightarrow 0$. In Ref. [3] we showed that the role of the factor x^{-a} in Eq. (3) is to mimic accounting for the total resummation of LL performed in Refs [4, 5]. Similarly to LL, the factor x^{-a} provides the steep rise to g_1 at small x and sets the Regge asymptotics for g_1 at $x \rightarrow 0$, with the exponent a being the intercept. The presence of this factor is very important for extrapolating DGLAP into the region **B**: When the factor x^{-a} is dropped from Eq. (3), DGLAP stops to work at $x \lesssim 0.05$ (see Ref. [3] for detail). Accounting for the LL resummation is beyond the DGLAP framework because LL come the phase space violating the base of DGLAP: the DGLAP -ordering

$$\mu^2 < k_{1\perp}^2 < k_{2\perp}^2 < \dots < Q^2 \quad (4)$$

for the ladder partons. LL can be accounted only when the ordering Eq. (4) is lifted and all $k_{i\perp}$ obey

$$\mu^2 < k_{i\perp}^2 < (p+q)^2 \approx (1-x)2pq \approx 2pq \quad (5)$$

at small x . Replacing Eq. (4) by Eq. (5) leads inevitably to the change of the DGLAP parametrization

$$\alpha_s^{DGLAP} = \alpha_s(Q^2) \quad (6)$$

by the alternative parametrization of α_s given by Eq. (13). This parametrization was obtained in Ref. [6] and was used in Refs. [4, 5] in order to find explicit expressions accounting for the LL resummation for g_1 in the region **B**. Obviously, those expressions invoke the fits for the initial parton densities without the singular factors x^{-a} . Let us note that replacement of Eq. (4) by Eq. (5) brings a more involved μ -dependence to g_1 . Indeed, Eq. (4) makes contributions of gluon ladder rungs be infrared (IR) stable, with μ acting as a IR cut-off for the lowest rung and $k_{i\perp}$ playing the role of the IR cut-off for the $i+1$ -rung. In contrast, Eq. (5) implies that μ acts as the IR cut-off for every rung.

Besides the regions **A** and **B**, it is necessary to know g_1 in the region **C**:

$$\mathbf{C}: \quad Q^2 < \mu^2, \quad x \ll 1 \quad (7)$$

because this region is studied experimentally by the COMPASS collaboration. Obviously, DGLAP cannot be exploited here. Alternatively, in Refs. [7, 8] we obtained expressions for g_1 in the region **C**. In Ref. [7] we showed that g_1 practically does not depend on x at small x , even at $x \ll 1$. Instead, it depends on the total invariant energy $2pq$. Experimental investigation of this dependence is extremely interesting because according to our results g_1 , being positive at small $2pq$, can turn negative at greater values of this variable. The position of the turning point is sensitive to the ratio between the initial quark and gluon densities, so its experimental detection would enable to estimate this ratio. In Ref. [8] we analyzed the power contributions $\sim 1/(Q^2)^k$ to g_1 usually attributed to higher twists. We proved that a great amount of those corrections have a simple perturbative origin and resummed them. Therefore, the genuine impact of higher twists can be estimated only after accounting for the perturbative Q^2 -corrections.

II. DESCRIPTION OF g_1 IN THE REGION **B**

The total resummation of the double-logarithms (DL) and single-logarithms of x in the region **B** was done in Refs. [4, 5]. In particular, the non-singlet component, g_1^{NS} of g_1 is

$$g_1^{NS}(x, Q^2) = (e_q^2/2) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega C_{NS}(\omega) \delta q(\omega) \exp(H_{NS}(\omega) \ln(Q^2/\mu^2)) , \quad (8)$$

with new coefficient functions C_{NS} ,

$$C_{NS}(\omega) = \frac{\omega}{\omega - H_{NS}^{(\pm)}(\omega)} \quad (9)$$

and anomalous dimensions H_{NS} ,

$$H_{NS} = (1/2) \left[\omega - \sqrt{\omega^2 - B(\omega)} \right] \quad (10)$$

where

$$B(\omega) = (4\pi C_F (1 + \omega/2) A(\omega) + D(\omega)) / (2\pi^2) . \quad (11)$$

$D(\omega)$ and $A(\omega)$ in Eq. (11) are expressed in terms of $\rho = \ln(1/x)$, $\eta = \ln(\mu^2/\Lambda_{QCD}^2)$, $b = (33 - 2n_f)/12\pi$ and the color factors $C_F = 4/3$, $N = 3$:

$$D(\omega) = \frac{2C_F}{b^2 N} \int_0^\infty d\rho e^{-\omega\rho} \ln\left(\frac{\rho+\eta}{\eta}\right) \left[\frac{\rho+\eta}{(\rho+\eta)^2 + \pi^2} \mp \frac{1}{\eta} \right] , \quad (12)$$

$$A(\omega) = \frac{1}{b} \left[\frac{\eta}{\eta^2 + \pi^2} - \int_0^\infty \frac{d\rho e^{-\omega\rho}}{(\rho+\eta)^2 + \pi^2} \right]. \quad (13)$$

H_S and C_{NS} account for DL and SL contributions to all orders in α_s . Eq. (13) and (12) depend on the IR cut-off μ through variable η . It is shown in Refs. [4, 5] that there exists an Optimal scale for fixing μ : $\mu \approx 1$ GeV for g_1^{NS} and $\mu \approx 5$ GeV for g_1^S . The arguments in favor of existence of the Optimal scale were given in Ref. [8]. Eq. (8) predicts that g_1 exhibits the power behavior in x and Q^2 when $x \rightarrow 0$:

$$g_1^{NS} \sim (Q^2/x^2)^{\Delta_{NS}/2}, \quad g_1^S \sim (Q^2/x^2)^{\Delta_S/2} \quad (14)$$

where the non-singlet and singlet intercepts are $\Delta_{NS} = 0.42$, $\Delta_S = 0.86$ respectively. However the asymptotic expressions (14) should be used with great care: According to Ref. [3], Eq. (14) should not be used at $x \gtrsim 10^{-6}$. So, Eq. (8) should be used instead of Eq. (14) at available small x . Expressions accounting the total resummation of LL for the singlet g_1 in the region **B** were obtained in Ref. [5]. They are more complicated than Eq. (8) because involve two coefficient functions and four anomalous dimensions.

III. DESCRIPTION OF g_1 IN THE REGION **C**

Region **C** is defined in Eq. (7). It includes small Q^2 , so there are not large contributions $\ln^k(Q^2/\mu^2)$ in this region. In other words, the DGLAP ordering of Eq. (4) does not make sense in the region **C**, which makes impossible exploiting DGLAP here. In contrast, Eq. (4) is not sensitive to the value of Q^2 and therefore the total resummation of LL does make sense in the region **C**. In Ref. [7] we suggested that the shift

$$Q^2 \rightarrow Q^2 + \mu^2 \quad (15)$$

would allow for extrapolating our previous results (obtained in Refs. [4, 5] for g_1 in the region **B**) into the region **C**. Then in Ref. [8] we proved this suggestion. Therefore, applying Eq. (15) to g_1^{NS} leads to the following expression for g_1^{NS} valid in the regions **B** and **C**:

$$g_1^{NS}(x+z, Q^2) = (e_q^2/2) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x+z} \right)^\omega C_{NS}(\omega) \delta q(\omega) \exp(H_{NS}(\omega) \ln((Q^2 + \mu^2)/\mu^2)) , \quad (16)$$

where $z = \mu^2/2pq$. Obviously, Eq. (16) reproduces Eq. (8) in the region **B**. Expression for g_1^S looks similarly but more complicated, see Refs. [7, 8] for detail. Let us notice that the idea of considering DIS in the small- Q^2 region through the shift Eq. (15) is not new. It was introduced by Nachtmann in Ref. [10] and used after that by many authors (see e.g. [11]), being based on different phenomenological considerations. On the contrary, our approach is based on the analysis of the Feynman graphs contributing to g_1 .

IV. PREDICTION FOR THE COMPASS EXPERIMENTS

The COMPASS collaboration now measures the singlet g_1^S at $x \sim 10^{-3}$ and $Q^2 \lesssim 1$ GeV², i.e. in the kinematic region beyond the reach of DGLAP. However, our formulae for g_1^{NS} and g_1^S obtained in Refs. [7, 8] cover this region. Although expressions for singlet and non-singlet g_1 are different, with formulae for the singlet being much more complicated, we can explain the essence of our approach, using Eq. (16) as an illustration. According to results of [5], $\mu \approx 5$ GeV for g_1^S , so in the COMPASS experiment $Q^2 \ll \mu^2$. It means, $\ln^k(Q^2 + \mu^2)$ can be expanded into series in Q^2/μ^2 , with the first term independent of Q^2 :

$$g_1^S(x+z, Q^2, \mu^2) = g_1^S(z, \mu^2) + \sum_{k=1} (Q^2/\mu^2)^k E_k(z) \quad (17)$$

where $E_k(z)$ account for the total resummation of LL of z and

$$g_1^S(z, \mu^2) = \langle e_q^2/2 \rangle \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/z)^\omega [C_S^q(\omega) \delta q(\omega) + C_S^g(\omega) \delta g(\omega)], \quad (18)$$

so that $\delta q(\omega)$ and $\delta g(\omega)$ are the initial quark and gluon densities respectively and $C_S^{q,g}$ are the singlet coefficient functions. Explicit expressions for $C_S^{q,g}$ are given in Refs. [5, 7]. The standard fits for δq and δg contain singular factors $\sim x^{-a}$ which mimic the total resummation of leading logarithms of x . Such a resummation leads to the expressions for the coefficient functions different from the DGLAP ones. After that the singular factors in the fits can be dropped and the initial parton densities can be approximated by constants:

$$\delta q \approx N_q \quad \delta g \approx N_g , \quad (19)$$

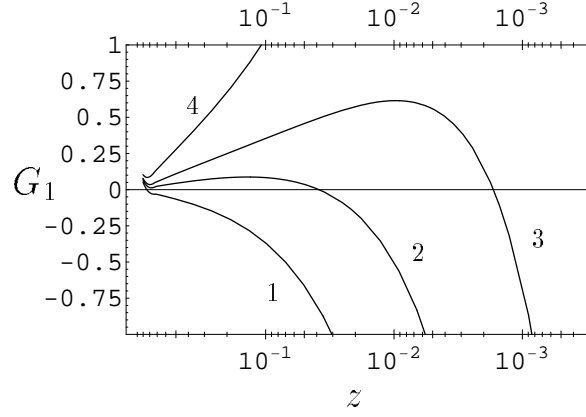


FIG. 1: G_1 evolution with decreasing $z = \mu^2/2(pq)$ for different values of ratio $r = \delta g/\delta q$: curve 1 - for $r = 0$, curve 2 - for $r = -5$, curve 3 - for $r = -8$ and curve 4 - for $r = -15$.

so, one can write

$$g_1(Q^2 \ll \mu^2) \approx (\langle e_q^2 \rangle / 2) N_q G_1(z) \quad (20)$$

with

$$G_1 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/z)^\omega [C_S^q + (N_g/N_q) C_S^g]. \quad (21)$$

Obviously, G_1 depends on the ratio N_g/N_q . The results for different values of the ratio $r = N_g/N_q$, G_1 are plotted in Fig. 1. When the gluon density is neglected, i.e. $N_g = 0$ (curve 1), G_1 being positive at $x \sim 1$, is getting negative very soon, at $z < 0.5$ and falls fast with decreasing z . When $N_g/N_q = -5$ (curve 2), G_1 remains positive and not large until $z \sim 10^{-1}$, turns negative at $z \sim 0.03$ and falls afterwards rapidly with decreasing z . This turning point where G_1 changes its sign is very sensitive to the magnitude of the ratio r . For instance, at $N_g/N_q = -8$ (curve 3), G_1 passes through zero at $z \sim 10^{-3}$. When $N_g/N_q < -10$, G_1 is positive at any experimentally reachable z (curve 4). Therefore, the experimental measurement of the turning point would allow to draw conclusions on the interplay between the initial quark and gluon densities.

V. REMARK ON THE HIGHER TWISTS CONTRIBUTIONS

In the region **B** one can expand terms $\sim (Q^2 + \mu^2)^k$ in Eq. (16) into series in $(\mu^2/Q^2)^n$ and represent $g_1^{NS}(x + z, Q^2, \mu^2)$ as follows:

$$g_1^{NS}(x + z, Q^2, \mu^2) = g_1^{NS}(x, Q^2/\mu^2) + \sum_{k=1} (\mu^2/Q^2)^k T_k \quad (22)$$

where $g_1^{NS}(x, Q^2/\mu^2)$ is given by Eq. (8); for explicit expressions for the factors T_k see Ref. [8]. The power terms in the rhs of Eq. (22) look like the power $\sim 1/(Q^2)^k$ -corrections and therefore the lhs of Eq. (22) can be interpreted as the total resummation of such corrections. These corrections are of the perturbative origin and have nothing in common with higher twists contributions ($\equiv HTW$). The latter appear in the conventional analysis of experimental data on the Polarized DIS as a discrepancy between the data and the theoretical predictions, with $g_1^{NS}(x, Q^2/\mu^2)$ being given by the Standard Approach:

$$g_1^{NS \text{ exp}} = g_1^{NSSA} + HTW. \quad (23)$$

Confronting Eq. (23) to Eq. (22) leads to an obvious conclusion: For estimating genuine higher twists contributions to g_1^{NS} , one should account, in the first place, for the perturbative power corrections predicted by Eq. (22); otherwise the estimates cannot be reliable. It is worth mentioning that we can easily explain the empirical observation made in the conventional analysis of experimental data: The power corrections exist for $Q^2 > 1 \text{ GeV}^2$ and disappear when $Q^2 \rightarrow 1 \text{ GeV}^2$. Indeed, in Eq. (22) $\mu = 1 \text{ GeV}$, so the expansion in the rhs of Eq. (22) make sense for $Q^2 > 1 \text{ GeV}^2$ only; at lesser Q^2 it should be replaced by the expansion of Eq. (16) in $(Q^2/\mu^2)^n$.

VI. CONCLUSION

Resummation of the leading logarithms of x is the straightforward and most natural way to describe g_1 at small x . Contrary to DGLAP, our approach is not sensitive to the value of Q^2 and allows one to describe g_1 at small x and arbitrary Q^2 in terms of the same expressions at large and small Q^2 . We have used it for studying the g_1 singlet at small Q^2 because this kinematic is presently investigated by the COMPASS collaboration. It turns out that g_1 in this region depends on $z = \mu^2/2pq$ only and practically does not depend on x , even at $x \ll 1$. Numerical calculations show that the sign of g_1 is positive at z close to 1 and can remain positive or become negative at smaller z , depending on the ratio between δg and δq . It is plotted in Fig. 1 for different values of $\delta g/\delta q$. Fig. 1 demonstrates that the position of the sign change point is sensitive to the ratio $\delta g/\delta q$, so the experimental measurement of this point would enable to estimate the impact of δg .

The alternative to the resummation is extrapolating DGLAP from its natural region of applicability (large x and large Q^2) into the region of small x and large Q^2 . As the DGLAP equations cannot account for the LL resummation, SA mimics the resummation through the special choice of the fits for the initial parton densities: the singular factors in the fits cause the steep rise of g_1 at small x and provide the Regge asymptotics for g_1 (however with the incorrect phenomenological intercepts) when $x \rightarrow 0$. They should be dropped when the total resummation of LL of x is taken into account. The remaining, regular x -terms of the DGLAP fits (the function φ in Eq. (3)) can obviously be replaced by much simpler expressions, so the number of phenomenological parameters in the fits can be reduced from 5 to 2 or even 1. To conclude, let us notice that extrapolating DGLAP into the small- x region, though provides a satisfactory agreement with experimental data, leads to various wrong statements. We enlisted the most of them in a recent Ref. [9]. Below we mention two more such wrong statements:

Statement 1: *The Q^2 -power corrections stem from higher twists g_1 and can be measured as the discrepancy between the DGLAP predictions and the data.* This statement is wrong as shown in the previous Sect.

Statement 2: *The impact of the LL resummation on the small- x behavior of g_1 is small.* This statement appears when the resummation has been included into the DGLAP expressions where the fits contain singular factors. Such inclusion is inconsistent and means actually a double counting of the LL contributions: once through the fits and secondly in the explicit way. It also affects the small- x asymptotics of g_1 , leading to the incorrect values of the intercepts of g_1 (see Ref. [3] for more detail).

VII. ACKNOWLEDGEMENT

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